Characteristics of Parker Print-Surf roughness as compared with Bekk smoothness*1

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ABSTRACT

What Parker Print-Surf (PPS) roughness means was discussed by comparing the theoretical equation to that of Bekk smoothness and by determining the wavelength range which PPS represents by the spectral analysis applied to the stylus profiles of paper. As a model of air flow through channels between paper surface and the metal measuring head of an air-leak type smoothness tester, the PPS model is more comprehensible than the Oken type (Bekk) model. However, what the testers survey is fundamentally common to the two methods. Thus, the two equations standing for the models are convertible and the conversion equation was led to be \( G_3 = 18.65 / \sqrt[3]{T_B} \), where \( G_3 \) is PPS roughness; \( T_B \) Oken type smoothness. The empirical data exhibited that the conversion equation applies to many papers except those with different compressibility. The correlation between PPS roughness and centerline average (Ra) at several cut-off wavelengths calculated from stylus surface profiles showed that PPS tester presses out paper surface so that the surface shape agrees with roughness at a cut-off wavelength of 234 µm for calendered handsheets. This cut-off wavelength gave the least sum of squared deviations from the theoretical conversion equation, \( G_3 = 2.13 \times Ra \). Oken type smoothness was considered to survey the longer wavelength components than PPS smoothness. However, the best-fitting cut-off wavelength was greatly

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dependent on the surface deformability of paper at a testing pressure in view of papermaking history and converting processes.

1. INTRODUCTION

Parker Print-Surf (PPS) roughness is sometimes called a kind of printing roughness because it correlates well to print quality. In recent publications overseas and also more recently in Japan, roughness of paper has been represented almost exclusively in terms of PPS roughness. It is explained by the fact that PPS was adopted as an ISO method in 1979.

PPS tester was invented in early 1970’s in Britain to improve Bekk and Sheffield smoothness testers. The principle is the air-leak method commonly among those three types. However, paper smoothness or roughness is expressed differently as time for a constant volume of air to leak out, passing between a paper surface and a metal plate clamping the sample for Bekk and Sheffield or as a mean gap calculated from the flow rate for PPS, respectively. PPS has additional improvements from the others regarding the clamping pressure and contact area between paper and the measuring head, as described in the following chapter.

In the present study, the theoretical differences between Oken type smoothness as an alternative of Bekk and PPS roughness were elucidated in the first place. Then, the wavelength PPS roughness corresponds to was determined based on the surface profiles by stylus profilometry using the spectral analysis.

2. EXPERIMENTAL

2.1 Specifications of Oken type and PPS testers

Table 1 lists different specifications between PPS[1, 2] and Oken type[3, 4] testers. The major ones are clamping pressure, shape of contacting part (land) of the measuring head with paper and unit of the expressed result. Oken type was basically designed to follow Bekk’s type so that every specification and measurement condition are equivalent to those of Bekk’s type except for the shape of the contacting part. Accordingly, the smoothness was given in
number of seconds and the smoothness value would be rather difficult to realize as indicating surface geometry of paper. In PPS tester, high enough pressures of paper clamping are used to simulate those of commercial printing practices. In the PPS mechanism, the measuring head consists of a land only 51 µm wide, which contacts paper and contributes to make a narrow channel across which air flow out. The width of the metering land is so small as to prevent air from flowing through inside the paper or leaking out from the backside.

2.2 Theoretical equations

Measured values by the two methods are expressed in different ways. Therefore, their equations must be carefully considered to compare what those values represent. Table 2 shows the individual equations. The mean gap $G_3$ in Equation (1) for PPS seems to be corresponding to the pore radius $r$ in Equation (2) for Oken type. So, the arrangement of Equation (2) with regard to $r$ and the substitution of the relationship $(V_B/T_B)/P_B = Q/D_P$ to unify the variables will give the following equation:

$$r = \left( \frac{8 \mu (V_B/T_B)}{\pi P_B} \right)^{\frac{1}{4}} = \left( \frac{8 \mu Q}{\pi \Delta P} \right)^{\frac{1}{4}}$$

Equation (2') represents that the pore radius $r$ of the model is proportional to the 4th root of $Q/D_P$. To put it another way, $r$ is inversely proportional to the 4th root of the number of seconds, that is, the Oken type smoothness. On the other hand, the mean gap $G_3$ of PPS is proportional to an inverse of a cube root of $Q/D_P$. Besides, they are different in the coefficient; $8/\pi$ for Oken type and 12 for PPS. This discordance stems from different modeling ideas.

2.3 Differences in equation derivation and modeling

Between PPS roughness and Oken type smoothness, the derived equation depends on what shape to regard a channel for air leak as. It is a slab for PPS or a bunch of circular tubes for Oken type as shown in Figure 1. Following is how to derive Equations (1) and (2).

2.3.1 PPS model

Figure 2 shows the channel between parallel plates, a laminar flow model which the PPS method assumes. Equation (1) for PPS is derived according to a general textbook[5]
describing details of fluid behavior. The Navier-Stokes equation for the $z$ component is

$$
p \left( \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left( \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) - \frac{\partial p}{\partial z} + \rho g_z \quad (3)
$$

By the substitutions of the continuity equation: $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$ and conditions such as $\frac{\partial v_z}{\partial t} = 0$ for steady state, $v_x = 0$, $v_y = 0$, $\frac{\partial v_z}{\partial z} = 0$, $\frac{\partial^2 v_z}{\partial z^2} = 0$, $\frac{\partial v_z}{\partial x} = 0$ and $\frac{\partial^2 v_z}{\partial x^2} = 0$, Equation (3) simply becomes

$$
\frac{\partial p}{\partial z} = \mu \frac{\partial^2 v_z}{\partial y^2} \quad (4)
$$

because $g_z = 0$ for the present case of a horizontal pipe.

$p$ is a function of neither $x$ nor $y$. Also, $\frac{\partial p}{\partial z}$ is constant since $v_z$ is not a function of $z$.

Then, Equation (4) becomes an ordinary differential equation.

$$
\frac{d^2 v_z}{dy^2} = \frac{\mu}{\Delta z} \frac{dp}{dz} = \text{const} \quad (5)
$$

Integrating Equation (5) twice using the conditions: $\frac{dv_z}{dy} = 0$ at $y = 0$ for symmetry and $v_z = 0$ at $y = y_0$,

$$
v_z = \frac{1}{2\mu} \frac{dp}{dz} (y^2 - y_0^2) \quad (6)
$$

Integrating this derivative, Equation (6), from $-y_0$ to $y_0$,

$$
Q = \int_{-y_0}^{y_0} w v_z dy = -\left. \frac{2wy_0^3}{3\mu} \frac{dp}{dz} \right|_{y_0} = \frac{wG_3^3 \Delta P}{12\mu b} \quad (7)
$$

where $G_3 = 2y_0$ and $\frac{dp}{dz} = -\Delta P / b$ as Figure 3 indicates necessary variables. This comes to Equation (1).

2.3.2 Oken type model

A channel through which air flow out is regarded as number $n$ of circular tubes with length $l$ and radius $r$ ($n=1$ assumed in the equation) in the Oken type model. The calculation[6] can be made also based on the Hagen-Poiseuille flow. Figure 4 illustrates air flow in a single circular tube.

Since $v_x$ and $v_y$ are zero, the continuity equation becomes $\frac{\partial v_z}{\partial z} = 0$. For steady state $\frac{\partial v_z}{\partial t} = 0$. Then, the Navier-Stokes equation for the $z$ component, Equation (3) becomes,
\[ \frac{dp}{dz} = \mu \left( \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial y^2} \right) \]  \hspace{1cm} (8)

To solve Equation (8), we can use cylindrical coordinates, giving
\[ z = z \quad x = r \cos \theta \quad y = r \sin \theta \quad r = \sqrt{x^2 + y^2} \quad \theta = \tan^{-1} \frac{y}{x} \]

Substituting these into Equation (8),
\[ \frac{1}{\mu} \frac{dp}{dz} = \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} \]
\[ \hspace{1cm} (9) \]

This coordinates conversion is a basic formula of partial differentiation[6].
\[ \frac{\partial^2 v_z}{\partial \theta^2} = 0 \] stands because of symmetry and \( dp / dz \) is constant. So, Equation (9) becomes,
\[ \frac{1}{\mu} \frac{dp}{dz} = \text{const} = \frac{d^2 v_z}{dr^2} + \frac{1}{r} \frac{dv_z}{dr} + \frac{1}{r} \frac{d}{dr} \left( \frac{dv_z}{dr} \right) \]
\[ \hspace{1cm} (10) \]

Using the conditions; \( dv_z / dr = 0 \) at \( r = 0 \) for the first integration and \( v_z = 0 \) at \( r = R \) (tube radius), Equation (10) becomes,
\[ v_z = \frac{1}{4\mu} \frac{dp}{dz} \left( r^2 - R^2 \right) \]

Volume velocity \( Q_B \) is obtained by integrating this equation from 0 to \( R \) and substituting \( dp / dz = -\Delta P / l \) in the following manner.
\[ Q_B = \int_0^R 2\pi r v_z dr = -\frac{\pi R^4}{8\mu} \frac{dP}{dz} = \frac{\pi \Delta P R^4}{8\mu} l \]

This is how to lead Equation (2).

2.3.3 Relationship between Oken type smoothness and PPS roughness

The detailed derivations mentioned above showed that the two methods differ in whether the gap between paper and the measuring head is regarded as a bunch of circular tubes or as a slab, although the principle is exactly common. In the Oken type model, radius \( r \) of pores can indicate roughness of paper. However, \( r \) cannot be calculated from a measured smoothness because the pore length \( l \) is also unknown. Therefore, it seems to be meaningless to bring in two unknown variables in one equation. In contrast, in the slab model for PPS, the mean gap \( G_3 \), namely PPS roughness, is an only variable and explicitly decided. Besides, centerline
average roughness ($Ra$) very often calculated from surface profiles refers to the mean distance in the thickness direction from the centerline. Thus, $Ra$ is expressed in the same unit with that of PPS roughness, that is, $\mu$m. Eventually, The slab model as for PPS is definitive and more comprehensible. However, no matter what model is assumed, the measurements are both based on the Hagen-Poiseuille flow between the gap. Therefore, Oken type smoothness and PPS roughness can be related using a conversion equation. The conversion equation can be derived this manner. The theoretical Equation (2) for Oken type comes from the following equation[4],

$$Q = \frac{\pi (P_c - P) R_0^4}{8 \mu L_0} = \frac{\pi P r^4}{8 \mu l}$$

(11)

This relation was obtained from that flow rate of air from the constant pressure chamber to the pressure-measuring chamber is equal to that from the pressure-measuring chamber to the outside. $R_0$ and $L_0$ are the radius and length of the tube connecting the two chambers, respectively. $P_B$ in Equation (2) was replaced with $P$ in Equation (11). Equation (1), by replacing $dP$ with $P$, becomes

$$Q = \frac{WG_3^3 P}{12 \mu b}$$

(1’)

$Q$ in both Equations (11) and (1’) is common so that the combination of the two equations gives

$$\frac{\pi (P_c - P) R_0^4}{8 \mu L_0} = \frac{WG_3^3 P}{12 \mu b}$$

(12)

where, $R_0$, $L_0$, $w$ and $b$ are 0.15 mm, 50 mm, 735 mm and 1 mm, respectively. Substitution of those known instrument constants into Equation (12) gives a simple interchangeable relationship.

$$G_3 = \frac{18.65}{3 \sqrt{b}}$$

(13)

This equation suggests that PPS roughness is proportional to an inverse of a cube root of Oken type smoothness. However, it must be noted that this relation could stand if the deformed surface shape was exactly the same under the measuring head between the two
types of tester. Practically, the measured values from these two testers would not satisfy this conversion equation even for the identical sheet of paper because of different conditions like clamping pressure and contact area.

2.3.3 Centerline average roughness and the air-leak methods

Centerline average roughness ($R_a$) calculated from a surface profile is a value of vertical distance. This means that $R_a$ should be directly related to PPS roughness expressing the mean gap, $G_3$ of the slab model because those two have the same dimension. Thus, $R_a$ should be related to an inverse of a cube root of Oken type smoothness according to equation (13) although an inverse of a forth root of Oken type smoothness was plotted in our previous paper[8] where a circular tube radius $r$ of Oken type model was considered to be the most indicative variable.

2.4 Experimental relation between Oken type and PPS

2.4.1 Sample preparation

2.4.1.1 Handsheets

Handsheets were prepared to be with various levels of roughness by calendering and beating. The sheets were subsequently supercalendered on a laboratory scale at different conditions regarding temperature, linear pressure and number of passes. Differently beaten pulp was also used for another series of handsheets. The chosen revolution on PFI mill was 0, 5000, 10000 and 15000 for softwood; 0, 5000, 10000 and 30000 for hardwood.

2.4.1.1 Rough handsheets

Very rough paper was purposely prepared by replicating sandpaper surfaces onto handsheets at wet-press. Square-shaped handsheets were made of never-dried hardwood bleached kraft pulp beaten to 5000 revolutions PFI mill. The replication was made according to the following procedures. A sheet of water-resistant sandpaper was inserted between a handsheet and a standard metal plate with the sand-filled side against the wireside of the handsheet after the couch process. The sandpaper kept held there during the subsequent wet-press and the ring-restraint drying. The sandpaper had five grades of roughness, #400,
600, 800, 100 and 1500. The larger the number, the smoother the surface.

2.4.1.1 Machine-made basepaper and its coated grade

Machine-made basepaper for coating (P0) having a basis weight of 96.1 g/m² was used. Its coated grade (P1) was prepared by applying coating color consisting of 100 parts of kaolin, 10 parts of SB-latex and 5 parts of starch to a coat weight of 5.8 g/m². Then, those two papers were supercalendered to different levels of roughness for another series of sample. **Table 3** shows the calendering conditions employed.

2.4.1.1 Commercial coated paper

Four kinds of commercial coated sheets were also used for the last series. They were light weight coated paper, coat paper (as one of the categories of coated paper according to Japan Industrial Standard), mat coated paper and cast coated paper.

2.4.2 Smoothness and roughness measurements

Oken type smoothness and PPS roughness were measured. Then, centerline average roughness (Ra) was calculated from the stylus profiles and correlated to the PPS roughness. Briefly, Ra means an mean distance from the centerline of a profile. The calculation procedures for Ra were detailed elsewhere[8-10]. The stylus profilometer used was SE-3 model, Kosaka laboratory Inc., Osaka, Japan. For PPS roughness, Parker Print-Surf tester PPS 78, H. E. Messmer ltd., Britain was used; for Oken type smoothness, Denso-Aspero Meter KY-5, Asahi Seiko Inc., Tokyo, Japan.

### 3. RESULTS AND DISCUSSION

3.1 Empirical comparison between PPS and Oken type

**Figures 5** and 6 show the empirical relationship between Oken type smoothness and PPS roughness for the handsheets and for the machine-made sheets, respectively. Oken type smoothness was plotted on an inverse of a cube root scale according to Equation (13). PPS roughness was measured with a soft backing at a pressure of 0.98 MPa (10 kgf/cm²). This
PPS pressure was 10 times that of Oken type. The straight dotted line drawn from the origin stands for the theoretical relation according to the conversion Equation (13). The entire view of all the scattering data points may provide an idea that they do not correlate well. But, if attention is paid to data points in the same series, some of them show an approximately linear relation, namely within the individual series which are the supercalendered softwood handsheets and supercalendered hardwood handsheets in Figure 5 and the supercalendered machine basepaper $P_0$, its coated paper $P_1$ and the commercial coated papers in Figure 6. Besides, the rough sheets due to replication of the sandpaper surfaces (denoted by gray triangles) in Figure 5 differ to a great deal in Oken type smoothness, but do not show as much difference as it from sample to sample in PPS roughness. It is presumably because rough surfaces were easily squeezed out in the PPS tester with a much larger clamping pressure than in Oken type tester. The slopes of each series other than the beating and sandpaper replica series were similar, but were shifted in parallel, probably because the surface compressibility varied with filler loading, beating and coating. Generally, beating increases fibers and sheet rigidity. In Figure 5, the remarkable effect of the rigidity increased by beating was observed with the overbeaten samples (denoted by a, hardwood to 30000 rev. PFI mill; b, softwood to 10000 rev.; and c, softwood 15000 rev.) as compared to others beaten to 5000 rev. normally. Those data points are shifted upward the Y-axis with beating; unbeaten pulps d and e downward. Beating-induced rigidity seems to make a paper surface less compressive. PPS is apt to be sensitive to surface compressibility because the clamping pressure is 10 times that of Oken type. This sensitivity, which is important in practical printings, should be a reason why people call PPS the printing roughness. Table 4 ensures this assumption. The reduction percentage of PPS roughness from at 0.49 MPa (5 kgf/cm²) to at 1.96 MPa (20 kgf/cm²) decreased with beating except for the most beaten sample. If the conversion Equation (13) is correct, it may be strange that there are some values of PPS roughness larger than the inverse of a cube root of Oken type smoothness for the identical sample. One possible reason for it is the hardness of the backing material. According to the standardized test methods, the backing
of Oken type tester consists of a rubber sheet 4 mm thick with a Shore Durometer hardness of 28 units backed with a sponge sheet 3 mm thick (only a 4 mm thick pad of rubber with a hardness of 39 units for real Bekk[11]), while the PPS soft backing is neoprene litho blanket with a hardness of 85 units[12]. Another possible reason is that the author waited for a reading to be displayed stable for a longer time for Oken type, which made the sample more compressed.

3.2 Calendering influence on paper surface profile

Before discussing relationship between PPS roughness and surface profile, here will be shown a typical influence of calendering on surface profiles observed with basepaper P0 and its coated paper P1. Table 3 lists conditions C0 to C5 under which those papers were calendered. Figures 7 and 8 show changes in Ra as a function of cut-off wavelength for paper P0 and P1, respectively. The smoothing effect on the paper surface by calendering can be seen clearly with Ra. The uncalendered sheets C0 had the largest values of Ra at larger values of cut-off wavelength both for P0 and P1. However, Ra of C0, as the cut-off wavelength decreased below about 200 µm, relatively became smaller than some calendered sheets. Flattening fiber surfaces by calendering seems to occur in a speckled manner and thus form hills with a flat top and sharp edges. Sharp edges as on calendered sheets would give rise to higher roughness at short cut-off wavelengths.

3.3 The relationship between PPS and surface profiles

The spectral analysis suggests how to relate PPS roughness to surface profiles. Surface profiles were measured with the handsheets as well as with paper P0 and P1 as already shown. It is our hypothesis that PPS roughness is supposed to correlate with Ra calculated from surface profiles at certain cut-off wavelength depending on tester attributes such as the clamping pressure and paper properties. Theoretically, supposed that the surface of paper in a tester is shaped sinusoidal as illustrated by Figure 9, the mean gap G3 in PPS would be

$$G_3 = \left\{ \frac{1}{2\pi} \int_{0}^{2\pi} \left(1 + \sin \theta \right)^{\frac{3}{2}} d\theta \right\}^{\frac{1}{3}} \approx 2.5^{\frac{1}{3}} = 1.36$$

(14)
and the $Ra$ would be

$$Ra = \frac{1}{2\pi} \int_0^{2\pi} \sin \theta \, d\theta = \frac{2}{\pi} = 0.64$$  \hspace{1cm} (15)$$

Thus, Equations (14) and (15) give the relationship,

$$G_3 = 2.13 \times Ra$$  \hspace{1cm} (16)$$

Figure 10 shows $Ra$ of stylus profiles at specified cut-off wavelengths as a function of PPS roughness (S10) for all the handsheets. The dotted line represents the theoretical relation, Equation (16). The beating series gave poor correlation between PPS roughness and $Ra$, while that of the calendering series except the two uncalendered sheet plots away from the others gave considerably good correlation. Turing to the numerical comparison, the plots of the calendered sheets came the closest on the theoretical relationship of Equation (16), the dotted line, at a cut-off wavelength of $234 \, \mu m$. This cut-off wavelength was determined so that the sum of squared deviations from the theoretical relationship except for the two plots for uncalendered ones became the least. Equation (16) was derived when paper surface was ideally assumed sinusoidal. More irregular and deeper depressions on a practical paper surface would make the coefficient of Equation (16) larger, thus reducing the best-fitting cut-off wavelength to well less than $234 \, \mu m$.

Figure 11 likewise shows $Ra$ as a function of Oken type smoothness. The inverse root is taken on the horizontal axis to be with the same unit dimension as PPS roughness according to Equation (13). Oken type smoothness and $Ra$ agreed best so that the plots fall the closest to the line of the theoretical relationship at a cut-off wavelength of $410 \, \mu m$. This cut-off wavelength was longer than that between PPS and $Ra$. It suggests that PPS surveys the components of shorter wavelengths of surface roughness than Oken type.

Table 5 summarizes the best-fitting cut-off wavelengths also for basepaper $P_0$ and its coated paper $P_1$. The best-fitting cut-off wavelength for $P_0$ was irregularly long because of grooves on the surface observed running in Machine direction. The stylus scanned the surface in parallel to the grooves and assessed the roughness less than the overall level. Papermaking
history and converting processes should change surface shape of paper so that deformation behavior in a air-leak roughness tester will be varied. Thus, best-fitting cut-off wavelength to correlate air-leak roughness to $Ra$ of surface profiles is difficult to determine in general to all kinds of paper.

4. CONCLUSIONS

As a model of channels for air flow between paper surface and the metal measuring head of a smoothness tester based on the air leak method, the PPS model is more comprehensible than the Oken type (Bekk) model. It is because the PPS model has a single variable, the mean gap to define the channel shape although the Oken type model has two variables, the pore length and pore diameter; one needs to be evaluated to determine the other. However, regardless of the model, what the testers survey is fundamentally common to the two methods. Thus, the two equations resulting from the different models are convertible and the conversion equation was led to be $G_3 = 18.65 / \sqrt[3]{T_B}$, where $G_3$ and $T_B$ are PPS roughness and Oken type smoothness. Commercial coated papers, machine-made basepaper for coating and variously calendered handsheets followed this relationship well; handsheets made of variously beaten pulps, very rough handsheets and machine-made basepaper for coating did not mainly because of the compressibility which acted differently in the tester at the different clamping pressures. The spectral analysis applied to stylus surface profiles resulted in that PPS tester presses out paper surface under the measuring head so that the surface shape agrees with centerline average roughness ($Ra$) at a cut-off wavelength of 234 $\mu m$ for calendered handsheets. This cut-off wavelength gave the least sum of squared deviations from the theoretical conversion equation, $G_3 = 2.13 \times Ra$. Oken type smoothness was considered to survey the longer wavelength components than PPS. However, the best-fitting cut-off wavelength was greatly dependent on the surface deformability of paper at a testing pressure in view of papermaking history and converting processes.
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和文要旨

「ベック法との比較による紙のパーカプリントサーフ粗さの特徴」

東京大学大学院農学生命科学系研究科生物材料科学専攻製紙科学研究室
江前敏晴、尾鍋史彦

パーカプリントサーフ(PPS)粗さがどのような紙の表面粗さを表しているかを考察した。PPS 粗さとベック平滑度の理論式の比較をまず行い、次に紙表面の触針式プロファイルのスペクトル解析から PPS が表している波長領域を求めた。理論式の比較から、紙表面と空気漏洩式平滑度計の金属ヘッドとの間にできる間隙を通る空気の流れのモデルとして、PPS モデルの方が王研（ベック）式モデルより合理的であることがわかった。しかし、両試験器が測定しようとするものは同一のものであるので、2つの理論式を関係づけることが可能であり、その変換式は \( G_3 = 18.65 / \sqrt{T_B} \) （ \( G_3 \): PPS 粗さ、\( T_B \): 王研式平滑度）と導くことができた。測定データの比較では、この変換式が多の紙について概ね成り立つが、叩解度のことなる紙など、圧縮性のことなるものについては成り立たなかった。

カレンダーがけした手抄き紙について、触針式プロファイルを様々なカットオフ値に設定して計算した中心線平均粗さ(\( Ra \))と PPS 粗さとの相関を調べたところ、紙の表面形状は PPS 試験器内で波長 234 μm に相当する粗さになるように変形することが示唆された。このカットオフ波長は、紙表面が正弦曲線であるとする仮定した場合の変換式 \( G_3 = 2.13 \times Ra \) からの偏差の 2 乗和を最小にするものである。同様に王研式平滑度は波長 410 μm に相当する粗さを見ていることが示唆された。一般的には、変換式に最も近い関係が成り立つカットオフ波長は、抄紙や加工の方法などによって変化する、紙表面の変形特性に依存すると考えられる。
Table 1 Major differences between Oken type and PPS testers

<table>
<thead>
<tr>
<th></th>
<th>Oken type</th>
<th>PPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clamping pressure, MPa</td>
<td>0.98 (1.0)</td>
<td>0.49, 0.98 or 1.96 (5, 10 or 20)</td>
</tr>
<tr>
<td>(kgf/cm²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of contact area</td>
<td>Nine concentric lands 1 mm wide each</td>
<td>A circular land 51 μm wide</td>
</tr>
<tr>
<td>Measured value</td>
<td>Time when air flows out (s)</td>
<td>Mean gap (μm)</td>
</tr>
</tbody>
</table>

Table 2 Comparison of the two theoretical equations

<table>
<thead>
<tr>
<th></th>
<th>PPS</th>
<th>Oken type (Bekk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_3 = \left( \frac{12 \mu b Q}{w \Delta P} \right)^{\frac{1}{3}} )</td>
<td>( T_B = \frac{8 \mu V_B l}{\pi P_B r^2} )</td>
<td></td>
</tr>
</tbody>
</table>

\( G_3 \) = mean gap  
\( \mu \) = viscosity of air  
\( b \) = distance through which air flows across metering land  
\( Q \) = volume of air flowing in unit time  
\( w \) = effective length of metering length  
\( \Delta P \) = pressure drop across metering land  

\( T_B \) = Oken type smoothness  
\( \mu \) = viscosity of air  
\( l \) = pore length  
\( V_B \) = air volume  
\( P_B \) = pressure drop  
\( r \) = pore radius
Table 3  Calendering conditions for machine-made basepaper P0 and its coated paper P1

<table>
<thead>
<tr>
<th>Condition</th>
<th>C0</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass, times</td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Temperature, °C</td>
<td>-</td>
<td>30</td>
<td>34</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Nip pressure, kN/m</td>
<td>-</td>
<td>39.2</td>
<td>58.9</td>
<td>58.9</td>
<td>98.1</td>
<td>144.2</td>
</tr>
</tbody>
</table>

Table 4  Compressibility of handsheets from variously beaten pulps. The sample clamping pressure is 0.49 MPa (5 kgf/cm²) for S5 and 1.96 MPa (20 kgf/cm²) for S20, respectively.

<table>
<thead>
<tr>
<th>Furnish</th>
<th>PFI mill count, revolution</th>
<th>PPS roughness, μm</th>
<th>change</th>
<th>change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>S5</td>
<td>S20</td>
<td>S20-S5</td>
</tr>
<tr>
<td>Hardwood</td>
<td>0</td>
<td>8.66</td>
<td>6.83</td>
<td>-1.83</td>
</tr>
<tr>
<td></td>
<td>5000</td>
<td>6.35</td>
<td>5.24</td>
<td>-1.11</td>
</tr>
<tr>
<td></td>
<td>10000</td>
<td>6.25</td>
<td>5.36</td>
<td>-0.89</td>
</tr>
<tr>
<td></td>
<td>30000</td>
<td>6.77</td>
<td>5.55</td>
<td>-1.22</td>
</tr>
<tr>
<td>Softwood</td>
<td>0</td>
<td>8.71</td>
<td>7.18</td>
<td>-1.53</td>
</tr>
<tr>
<td></td>
<td>5000</td>
<td>7.99</td>
<td>7.22</td>
<td>-0.77</td>
</tr>
<tr>
<td></td>
<td>10000</td>
<td>9.11</td>
<td>8.28</td>
<td>-0.82</td>
</tr>
<tr>
<td></td>
<td>15000</td>
<td>9.24</td>
<td>8.29</td>
<td>-0.95</td>
</tr>
</tbody>
</table>

Table 5  Best-fitting cut-off wavelengths to correlate centerline average roughness (Ra) and air leak roughness or smoothness

<table>
<thead>
<tr>
<th>Paper</th>
<th>Best-fitting wavelength, μm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PPS roughness</td>
</tr>
<tr>
<td>Handsheets calendered</td>
<td>234</td>
</tr>
<tr>
<td>Basepaper P0</td>
<td>&gt; 4096</td>
</tr>
<tr>
<td>Coated paper P1</td>
<td>410</td>
</tr>
</tbody>
</table>
Fig. 1 Comparison of channel models between PPS and Oken type.

Fig. 2 Flow between two parallel plates as a model of PPS.
Fig. 3 Schematic diagram of a channel for air leak in the PPS model.

Fig. 4 Horizontal flow in a tube as a model of Oken type.
Fig. 5: Relationship between PPS roughness and Oken type smoothness for handsheets.

Fig. 6 Relationship between PPS roughness and Oken type smoothness for machine-made sheets. Hard backing used instead for coated paper P1.
Fig. 7 Calendering effect on centerline average roughness with decreased cut-off wavelength for basepaper P0.
Fig. 8 Calendering effect on centerline average roughness with decreased cut-off wavelength for coated paper P1.
Fig. 9  Schematic diagram of a gap between paper and the measuring head when surface profile is assumed to be sinusoidal.
Fig. 10 Relationship between centerline average roughness ($Ra$) and PPS roughness. The length in individual figure represents a wavelength above which the wave component was cut-off in $Ra$ calculation.
The length in individual figure represents a wavelength above which the wave component was cut-off in $Ra$ calculation.

Fig. 11  Relationship between centerline average roughness ($Ra$) and Oken type smoothness.